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(54) HIGH-PERFORMANCE SEQUENCE ESTIMATION SYSTEM AND METHOD OF OPERATION

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See application file for complete search history.

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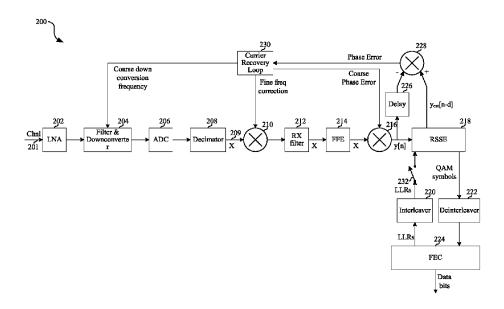
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(57) ABSTRACT

An electronic receiver comprises sequence estimation circuitry operable to implement a sequence estimation algorithm. In the sequence estimation algorithm, each of a plurality of possible current states of the signal may have associated with it a respective N_c possible prior states and a respective M state extensions, where N_c and M are integers greater than 1. Each iteration of the sequence estimation algorithm may comprise extending each of the plurality of possible current states of the signal by its respective N_c possible prior states and its respective M state extensions to generate a respective N_xM extended states for each of the plurality of possible current states. Each iteration of the sequence estimation algorithm may comprise, for each of the plurality of possible current states of the signal, selecting M of the respective N_a×M extended states to be state extensions for a next iteration of the sequence estimation algorithm.

19 Claims, 3 Drawing Sheets



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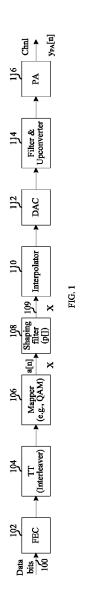
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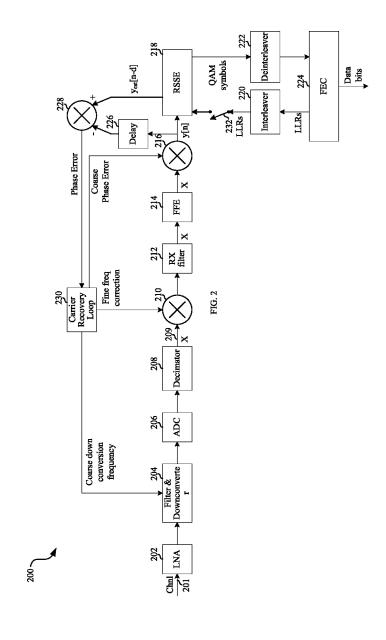
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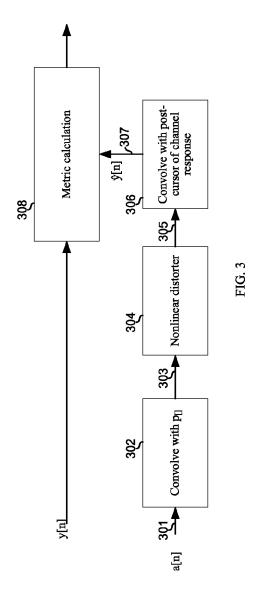
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HIGH-PERFORMANCE SEQUENCE ESTIMATION SYSTEM AND METHOD OF OPERATION

BACKGROUND

Limitations and disadvantages of conventional methods and systems for electronic communication will become apparent to one of skill in the art, through comparison of such approaches with some aspects of the present method and system set forth in the remainder of this disclosure with reference to the drawings.

BRIEF SUMMARY

Methods and systems are provided for communication system with high tolerance of phase noise and nonlinearity, substantially as illustrated by and/or described in connection with at least one of the figures, as set forth more completely in the claims.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 depicts a transmitter in accordance with an example implementation of this disclosure.

FIG. 2 depicts a receiver in accordance with an example implementation of this disclosure.

FIG. 3 depicts an example Viterbi implementation of the sequence estimation circuitry of FIG. 2.

DETAILED DESCRIPTION

As utilized herein the terms "circuits" and "circuitry" refer to physical electronic components (i.e. hardware) and any software and/or firmware ("code") which may configure the 35 hardware, be executed by the hardware, and or otherwise be associated with the hardware. As used herein, for example, a particular processor and memory may comprise a first "circuit" when executing a first one or more lines of code and may comprise a second "circuit" when executing a second one or 40 more lines of code. As utilized herein, "and/or" means any one or more of the items in the list joined by "and/or". As an example, "x and/or y" means any element of the three-element set $\{(x), (y), (x, y)\}$. In other words, "x and/or y" means "one or both of x and y." As another example, "x, y, and/or z" 45 means any element of the seven-element set $\{(x), (y), (z), (x, y), (y, z), ($ y), (x, z), (y, z), (x, y, z). In other words, "x, y and/or z" means "one or more of x, y and z." As utilized herein, the terms "e.g.," and "for example" set off lists of one or more non-limiting examples, instances, or illustrations. As utilized 50 herein, circuitry is "operable" to perform a function whenever the circuitry comprises the necessary hardware and code (if any is necessary) to perform the function, regardless of whether performance of the function is disabled, or not enabled, by some user-configurable setting.

A communication system in accordance with an example implementation of this disclosure may use a single-carrier air interface based on faster than Nyquist coded modulation. The signal processing in the system may be tailored for achieving high capacity by handling non-linearity and phase noise. The 60 system may be particularly suitable for cases of high-order transmission constellations (e.g. 1024QAM) where both power amplifier non-linearity and phase noise are significant.

An M-algorithm based reduced state sequence estimation (RSSE) architecture may be used as a near maximum likelihood receiver for such a communication system. The downside with M-algorithm architecture, however, is that it

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requires sorting multiple survivor hypothesis and therefore has a bottleneck (sorting) that limits achievable speedup by parallelization. An alternative approach to RSSE is the use of the Viterbi algorithm, which is actually a true maximum likelihood solution. However the number of states (order of complexity) of full Viterbi algorithm is A^{N_h-1} , where the signaling constellation size is denoted by A and the combination of pulse and channel duration (memory) is denoted by N_h . For a communication system in accordance with an example implementation of this disclosure, where A=64 and $N_h=24$, the resulting full Viterbi state count $A^{N_h-1}=64^{23}$ is huge. Such complexity may be reduced through use of a truncated Viterbi algorithm. However, in a communication system in accordance with an example implementation of this 15 disclosure, truncating the Viterbi to memory of N_t=6 symbols still results in a huge state count of $A^{N_r-1}=64^5$, which is not commercially feasible for some applications.

A set partitioning scheme (i.e. to divide the received state bits to N_c cosets and into parallel transitions) may be used to further reduce the number of Viterbi algorithm states needed. In one example implementation, per each of the N_t -1 symbols corresponding to a Viterbi state, each symbol's in-phase and quadrature LSBs are used to define n_c =4 cosets and the symbol's higher bits (in integer mapping) are parallel transitions, thus reducing the state count to N_c $^{N_t-1}$ =4 5 which is difficult but feasible for many applications.

The performance of Viterbi algorithm approach is limited due to limited memory (e.g. N_t =6 in the example described above). To further improve performance, this approach may 30 be augmented using multiple survivors per Viterbi state. This approach is referred to in this disclosure as the "Hybrid Viterbi M-Algorithm Approach" (or "M-Viterbi," for short). In an example implementation, a communication system uses Viterbi Memory of $N_t=3$ (i.e. $N_c^{N_t-1}=4^2$ Viterbi states), but allows at each Viterbi state multiple (e.g. M=16) survivors paths as used in the M-Algorithm. These Viterbi state extensions are referred to in this disclosure as "tails". Since the number of survivors required at each Viterbi state is lower than the number of survivors required for pure M-Algorithm, the sort bottleneck is resolved (since the M algorithm has to sort all survivors whereas M-Viterbi only has to sort survivors per state). Moreover, comparing at equal complexity, this approach achieves consistently better performance than the Viterbi Algorithm, and more significantly better performance than M-Algorithm at low SNR's (where the M-Algorithm occasionally loses the correct path and therefore is prone to long error events).

An example implementation of a M-Viterbi receiver in accordance with this disclosure may manage non-linearity and phase noise on top of inter-symbol-interference (ISI) due to channel and faster than Nyquist signaling. Faster than Nyquist signaling, with significant spectral compression (i.e. twice the signal BW), may be used to manage non-linearity. Since distortion BW is higher than signal BW, the M-Viterbi RSSE state may be updated at a multiple of original BW.

In a receiver in accordance with an example implementation of this disclosure, the power amplifier output, rather the transmitted symbols (accompanied by some post cursor ISI response), are reconstructed at the output of the FFE. Reconstructing the PA output rather than the transmitted symbols exposes nonlinear distortion and enables compensating for the nonlinear distortion using a maximum likelihood based approach.

In an example receiver in accordance with an implementation of this disclosure, the M-Viterbi Algorithm decodes a double convolution distorted response. That is, original symbols are first convolved by the response of the transmit pulse

shaping filter, then non-linearly distorted, and finally convolved by the post cursor of the channel response. In such an implementation, the FFE is used to convert the channel response to a response having a short post cursor portion and having most of its energy at the initial tap. This may, reduce 5 the probability of parallel transition error, and result in there being relatively little energy at symbol times beyond the depth of the Viterbi memory (NO.

In the "Viterbi" approach, the Viterbi RSSE is used to maintain a metric per the state, while in an example implementation using the "M-Viterbi" approach, a metric is maintained per pair of (state, tail). In both cases that metric is subsequently used to select the best sequence of states (i.e. survivor via traceback). The sequence of states by itself corresponds to the sequence of symbol cosets along the selected path. To update the state metric, the M-Viterbi RSSE uses the original state symbol history and carrier phase estimation. The parallel transitions (most significant bits (MSBs)) are determined per Viterbi state based on that particular state symbols history and carrier phase estimation. The state history allows to anticipate the nonlinear distortion (typically dealing with power amplifier non-linearity) while carrier phase estimation allows to anticipate the phase noise.

FIG. 1 depicts an example transmitter in accordance with an implementation of this disclosure.

Although the transmitter may use coded modulation, in order to achieve very low BER, it may use an additional outer FEC encoding circuit 102.

In the example transmitter 100, the outer FEC encoder 102 is followed by an interleaver circuit 104 and QAM mapper 30 circuit 106 that outputs symbols denoted as $a_{[n]}$. The QAM mapper 106 operates at faster baud rate than a conventional QAM system using the same bandwidth. In this regard, in an example implementation, if the transmitter 100 is allocated a bandwidth of W_0 for transmitting the data 100, then the QAM 35 mapper 106 uses a baud rate (BR)>W₀. For comparison, a conventional Nyquist-rate QAM system using the same bandwidth of W_0 and having excess bandwidth of β would use a band rate of $W_0/(1+\beta)$. In an example implementation, BR may be double the conventional band rate (i.e. BR= $2 \cdot W_0 / (1 + 40)$ β). The shaping filter circuit 108, characterized by an expression p_{1} , is run at the baud BR and is used to limit transmitter spectrum according to applicable spectral mask (e.g., specified by a standards and/or regulatory body). In an example implementation the shaping filter 108 bandwidth is lower 45 than baud rate. In an example implementation, the shaping filter 108 is part of modulation code and is designed to optimize coding gain. The output 109 of the shaping filer 108 is interpolated by interpolator circuit 110, converted from digital to analog by digital-to-analog converter (DAC) circuit 50 112, upconverted to carrier frequency by filter and upconverter circuit 114, and amplified by the PA 116, resulting in signal $y_{PA[n]}$, which is sent over a wireline or wireless channel.

FIG. 2 depicts a receiver in accordance with an example 55 implementation of this disclosure. In the receiver 200, the signal 201 (the result of signal $y_{PA[n]}$ passing through the channel) is received by one or more antennas or ports, amplified by an LNA 202, down-converted and filtered by filtering and downconversion circuit 204, sampled by analog-to-digital converter (ADC) circuit 206 and decimated by decimator circuit 208 resulting in signal 209. A carrier recovery loop circuit 230 is used both for driving the downconversion analog phase locked loop (PLL) (frequency conversion) of circuit 204 and a digital fine frequency correction via mixer 210. The 65 carrier recovery loop 230 is fed by a phase error derived by comparing, in circuit 228, the Viterbi/M-Viterbi algorithm

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delayed input signal $y_{[n-d]}$ from delay element **226** and the Viterbi/M-Viterbi estimated signal $y_{est_{[n-d]}}$ from RSSE circuit **218** (The delay d is to compensate for Viterbi/M-Viterbi processing delay and a short traceback used). In order to quickly track phase changes the traceback used for driving the carrier recovery loop may be smaller than that used for symbol demodulation.

After being rotated by circuit **210** based on the fine frequency correction output by carrier tracking loop **230**, the signal **209** is filtered by RX filter circuit **212** and processed by FFE circuit **214** and coarsely phase corrected in circuit **216** resulting in signal $y_{[n]}$ being input to RSSE **218**. The objective of the FFE is to recover PA output samples $y_{PA}[n]$ (instead of, or in addition to, recovering transmitted symbols $a_{[n]}$). The FFE **214** may adapt to minimize pre-cursor ISI, while being allowed to produce post cursor ISI. The output of the FFE is fed to the Viterbi/M-Viterbi based RSSE circuit **218** that, at the same time, decodes the modulated signal and manages post cursor ISI, h_{PC} , generated by the channel and FFE.

post cursor ISI, $h_{PC[n]}$, generated by the channel and FFE. Allowing the FFE **214** to output a signal with post-cursor ISI, $h_{PC[n]}$, where $h_{PC[n]}$ is the residual post cursor channel response not handled by the FFE **214**, along with using the Viterbi/M-Viterbi based RSSE circuit **218** to handle the post-cursor ISI may increase complexity of the RSSE **218**. Accordingly, in an example implementation where the channel is flat (within determined tolerances) the FFE circuit **214** may be constrained from generating significant post cursor ISI and/or the Viterbi/M-Viterbi-based search algorithm may be configured to expect a pulse that includes the post-cursor ISI. This combined filter is called the composite $h_{[n]} = h_{PC[n]} *p_{[n]}$ (where * denotes convolution).

The output soft-decisions (e.g., log likelihood ratios (LLRs)) of the Viterbi/M-Viterbi-based RSSE circuit 218 are fed to the de-interleaver circuit 222 and outer FEC decoder circuit 224 (for the initial iteration the LLR switch 232 is open, thus inputting zeroes to the RSSE circuit 218). The FEC decoder 224 may output the data bits at this stage. Alternately, to further improve performance, the receiver may perform additional iterations between the RSSE circuit 218 and the outer FEC decoder 224. For these iterations the LLR switch 232 is closed, and the LLR values are converted to extrinsic LLRs by subtracting the respective decoder LLRs input to RSSE circuit 218 from interleaver 220.

The Viterbi/M-Viterbi algorithms have two functions: (1) Equalizing the channel by handling the post cursor ISI; and (2) decoding the received signal. These two tasks may be performed at the same time for the equalizing function to be provided with the decoded decisions. In an example implementation, the receiver **200** may comprise a decision feedback equalizer. In cases of relatively flat/short channel, however, the receiver may disable the DFE (or not have a DFE at all) and may incorporate the DFE into the composite response $h_{[]}$, which is described below.

An MLSE Viterbi algorithm models the set of all possible transmissions using a Hidden Markov Model directed graph that is referred to as the "Trellis". Where the hidden states are the transmitted symbol indices $a_{[n]} \in \{0 \ldots A-1\}$ and the visible information is the conditional expectancy of the received signal denoted here as $y_{est[n]}$ (conditioned on previous and next trellis states). The trellis has a trellis root node and a trellis terminal node, and between them multiple columns of graph nodes. Each column corresponds to a transmission symbol time denoted by n. Graph nodes populating column n correspond to all possible hidden states of the transmission at that symbol time. The set of possible states per column n is called the state space and denoted by the set $s_{[n]}$. Directed graph edges, called branches, exist between two

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states corresponding to successive columns, between trellis root node and first column, and between last column and trellis terminal node. Every trellis path starting at the trellis root and ending at the trellis terminal node corresponds to a valid transmission, where the set of symbol indices along that path $a_{[n]}$ are mapped to actual symbol $\mu(a_{[n]})$ using the mapping function $\mu(a)$. The branches are indexed here using I_{br} . where the root and terminal states of a branch I_{br} are denoted $\mathbf{s}_{root}\left(\mathbf{I}_{br}\right)$ and $\mathbf{s}_{term}(\mathbf{I}_{br}),$ and the set of branches starting at a specific state s_0 are denoted $B(s_0)$. Every branch I_{br} is labeled by a deterministic value that corresponds to conditional expectancy of the received signal, and based deterministically on $\mathbf{s}_{root}(\mathbf{I}_{br})$ and $\mathbf{s}_{term}(\mathbf{I}_{br})$ states. In the MLSE case, the state space corresponds to all $A^{N_{t-1}}$ sequences of N_{t} -1 last symbol indices that, in conjunction with current symbol index $a_{[n]}$, provide a sufficiently good estimate of received signal $y_{est[n]}$ (i.e. the conditional expectancy). For this to happen N, must be large enough such that relative tail energy of the channel (including transmission pulse) is low relative to total channel 20 impulse response energy.

Reducing the state space can reduce complexity and memory requirements of a Viterbi decoder. This may be achieved by both reducing N_t significantly below the full channel duration N_h , and also using coset representation of each symbol index $a_{[n]} \in \{0 \dots A-1\}$. That is, given a set partitioning of the symbol constellation, the state space represents only the coset index $I_{cs}=a_{[n]}\%$ N_C instead of full symbol index $a_{[n]}$. In this case the coset index I_{cs} refers to least significant bits (LSBs) of the symbol index which are protected by the Trellis, while the parallel transitions corresponding to MSBs are not protected by the trellis. To map the symbol coset index I_{cs} and parallel transition index I_{ms} index we use the known mapping functions $\mu(I_{cs},I_{ms})$ that use set partitioning.

Viterbi State Space:

In an example implementation in which the RSSE circuit 218 uses the Viterbi algorithm, the RSSE circuit 218 maintains a set of states that corresponds to all possible symbols coset sequences of length N_t -1. In an example implementation, such as described above, the length of the sequences may be N_t-1=5. Every Viterbi state represents an infinite set of symbol vectors. Such that symbol vectors are partitioned according to the LSBs (cosets) of their latest N_r-1 symbols. For example, at symbol n, the symbol indices sequence 45 $\{a_{[n-k]}\}_{k=0}^{\infty}$ is represented by the following coset sequence

$$\begin{split} s_{[n]} &= \{a_{[n]} \% N_{c'} a_{[n-1]} \% N_{c'} a_{[n-2]} \% N_{c'} a_{[n-2]} \% \\ N_{c'} \dots, a_{[n-N_{p^{k-1}}]} \% N_{c}\}, \end{split} \tag{1}$$

where % denotes modulo, and N_C is the number of cosets 50 (which may also vary according to delay, i.e. by $N_{c_{[k]}}$). If the RSSE circuit **218** looks far enough into the past $(k \rightarrow \infty)$, there are infinitely many symbol sequences represented by the same small (in our example of size N_t -1=5) coset vector.

For each state $0 \le m \le 4^{N_r-1} - 1$ (representing a short coset 55 vector), the Viterbi memory of RSSE circuit 218 holds an accumulated metric of the state M; a history of N_h symbols $a[n], a_{[n-1]}, \dots a_{[n-N_h+1]};$ and a last phase estimation $\theta_{[n]}$. Hybrid M-Viterbi State Space

In an example implementation in which the RSSE circuit $\,$ 60 218 uses the M-Viterbi algorithm, up to M_{tail} different tails may be held for each Viterbi state. For each tail, the RSSE circuit 218 may keep and maintain: an accumulated metric of the state M; a history of N_h symbols $a_{[n]}, a_{[n-1]}, \ldots, a_{[n-N_h+1]};$ and a last phase estimation $\theta_{[n]}$. The RSSE circuit **218** may alternatively maintain a partial symbol history, e.g. a[n-D], $a_{[n-D-1]}, \dots a_{[n-N_h+1]}, \text{ where D>0, in which case } a_{[n]}, a_{[n-D+1]}$

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may be determined from the input data when extending the survivor (e.g. using multi-dimensional slicing).

Unlike the Viterbi algorithm, the M-Viterbi algorithm may experience duplicate tails (as does the M-Algorithm) per Viterbi state. Thus, the M-Viterbi algorithm requires a mechanism for pruning these duplicate tails for the same Viterbi state. Duplicate tails correspond to identical symbol history $a[n], a_{[n-1]}, \dots a_{[n-N_h+1]}$. When the RSSE circuit **218** detects duplicate tails, the tail(s) having worse metric (i.e. higher numerical value) may be discarded. This process may occur far slower than symbol rate, which is beneficial for complexity reduction.

Thus, whereas the Viterbi algorithm maintains a single previous Viterbi state for each current state, the M-Viterbi algorithm holds M different possible previous states for each current state. The tails captures state history that would be too old to be captured by a Viterbi trellis (i.e., the M-Viterbi captures state history that is longer than Viter imemory). The tails of the M-Viterbi algorithm efficiently describe a small subset of the possible long survivors, whereas the Viterbi trellis describes all possible short survivors and the M algorithm holds only M paths that do not include all possible short paths. Since short paths only contain information about most recent symbols (which is least reliable) the M-Viterbi reduces the probably of losing the correct path, as compared to the M-Algorithm.

Viterbi State Connectivity/Branches

When receiving at time n a new sample [n], the RSSE 218 may update all the Viterbi states from time corresponding to symbol n-1 to time corresponding to symbol n. For each possible state $s_{[n]}$ at time n, the RSSE circuit 218 examines all possible prior states $\mathbf{s}_{[n-1]}$ at time n-1 from which $\mathbf{s}_{[n]}$ could have possibly originated. In an example implementation, the set of prior states for $s_{[n-1]}$ is:

$$s_{[n-1]} = \{a_{[n-1]}\%N_{c}, a_{[n-2]}\%N_{c}, a_{[n-3]}\%\\N_{c} \dots a_{[n-N_{l}]}\%N_{c}\}. \tag{2}$$

Note that $a_{[n-1]}\%$ N_c , $a_{[n-2]}\%$ N_c , . . . , $a_{[n-N,+1]}\%$ N_c are common to $s_{[n]}$ and $s_{[n-1]}$ and therefore only to the property of algorithm decision. Basically these correspond to the narrowing of the set of possible prior states from which each particular $s_{[n]}$ may have originated. The different options for the oldest coset $a_{[n-N_i]}\%$ N_c complete the $s_{[n-1]}$ definition. Since coset $a_{[n-N_i]}\%$ N_c is not defined by $s_{[n]}$, there are N_c different possible prior states (possible values of $s_{[n-1]}$). Each such directed pair of states $s_{[n-1]} \rightarrow s_{[n]}$, where $s_{[n-1]}$ is the prior state and $s_{[n]}$ is a branch.

M-Viterbi State Additional Connectivity

In the M-Viterbi approach the RSSE circuit 218 considers, per such prior state $s_{[n-1]}$, all the M_{tails} possible tails. Each such tail is a possible sequence of symbol indices that ended in that state $s_{[n-1]}$ (i.e. a survivor). Thus, per $s_{[n]}$ state, we get M_{tails}·N_c survivors that are candidates that may have preceded that $s_{[n]}$ state. We denote each survivor at time n-1 as the pair (s[n-1], m), where s[n-1] is the prior state and m=1, 2, ..., M_{tails} is a tail associated with that prior state. An extended branch \mathbf{I}_{ebr} can then be defined as the directed pair of $(s[n-1], m) \rightarrow s[n]$ that associates with the new s[n] state, with possible prior survivor tail, belonging to the s[n-1] state.

Parallel Transitions (MSBs)

Similarly, the MSBs of the newest symbol $a_{[n]}$ (i.e. floor $(a_{[n]}/N_c)$), are not defined by $s_{[n]}$. Therefore for each state

$$\begin{split} S[_{n-1}] &= \{ a[_{n-1}] \% N_{c} a[_{n-2}] \% N_{c} a[_{n-3}] \% \\ N_{c} \dots a[_{n-N_{l}}] \% N_{c} \} \end{split} \tag{3}$$

(and in the M-Viterbi, for each of s[n-1] constituent tails m=1, 2, ..., M), we have several options for the value of the

MSBs (floor($a_{[n]}/N_c$) options, to be exact). These different options the values of MSBs do not amount to different branches, since the same set of MSBs may correspond to any branch $s_{[n-1]} \rightarrow s_{[n]}$. Instead, these different options are parallel transitions. In a conventional Viterbi decoder, the parallel transitions (i.e. $a_{[n]}$ MSBs) are based on $y_{[n]}$ and protected only by the decoder having determined these cosets. In contrast, in an example implementation of this disclosure, the RSSE circuit **218** protects the MSBs selection based on [n], $y_{[n-1]}$, and their cosets. In another example implementation of this disclosure, the RSSE circuit **218** first decodes the cosets, and then runs the Viterbi Algorithm or the Hybrid M-Viterbi again to decode the MSBs. In this second run, the cosets (LSBs) are fixed (to their decoded result from first run) thus allowing the RSSE circuit **218** to handle the MSBs.

(Coset) Viterbi Update

Every symbol time n, an example implementation of the RSSE circuit **218** using the Viterbi algorithm updates the metric for each state $\mathbf{s}_{[n]}$ based on the incoming branches (\mathbf{I}_{br}) and new received sample $\mathbf{y}_{[n]}$. The metric of each branch is minimized over possible parallel transitions (\mathbf{I}_{ms}) , and is then used to compute the following state \mathbf{s}_{new} metric

$$\begin{split} M(s_{new}) &= \\ & \operatorname{argmin}_{I_{br} \in B(s_{new})} \underset{I_{ms}}{\operatorname{argmin}} \|y_{[t]} - \zeta(s_{new}, s_{root}(I_{br}), I_{ms}\|^2 + M(s_{old}(I_{br})) \end{split}$$

where $B(s_{new})$ is the set of N_c possible incoming branches $s_{noor[k]} \rightarrow s_{new}$ to the state s_{new} ; I_{br} is a branch index; there are N_c possible incoming branches for s_{new} ; and $s_{root}(I_{br})$ is the root states of the branch I_{br} , which includes all the recent state history $\{a_{[n-k]}\}_{k=D}^{N_b}$ that is relevant for computing metric ("filter memory" and phase), D>0 where it is desired to maintain partial symbol history as explained above; I_{ms} is a parallel stransitions index, of which there are A/N_c possibilities; and $\zeta(s_{old}(I_{br}), I_{br}, I_{ms})$ is a predictor for $y_{[n]}$ based on $s_{new}, s_{old}(I_{br})$, I_{ms} , I_{ms}

Thus the state metric for state $s_{[n]}$ is taken as the minimum of a set of different branches $I_{br} \in B(s_{[n]})$, at the same time the RSSE circuit **218** stores the selected branch (providing the minimal metric in the formula above). The index of selected branch is stored in traceback memory of the RSSE circuit **218** that indicates for each state $s_{[n]}$ the selected root state $s_{[n-1]}$ (i.e. selected branch) and also the transmitted symbol $a_{[n]}$ associated with the transition to terminal state $s_{[n]}$. Note that the state index itself implies the coset $a_{[n]} \% N_c$ (i.e. the LSBs). Thus, the RSSE circuit **218** may only incrementally store the MSBs per state. Also note that traceback memory may not hold traceback data relating to very old information that exceeds the traceback depth discussed below. Thus, the traceback memory may be implemented as a cyclic buffer of depth at least as big as the traceback depth.

M-Viterbi Update

Every symbol time n, an example implementation of the RSSE circuit **218** using the M-Viterbi algorithm computes a 5s set of M_{tails} tails for each state s[n] based on the incoming $N_c \cdot M_{tails}$ extended branches (I_{ebr}) , and based on received sample y[n]. For each new state s[n], there are N_c prior states $\{[n-1]\}$ and for each such prior state there are M_{tails} possible tails that correspond to different symbol histories. Thus, in 60 total there are $N_c \cdot M_{tails}$ candidate (state, tails) pairs that may have preceded that new state. From this set the RSEE circuit **218** using the M-Viterbi algorithm selects a subset of candidates consisting of the best (e.g., having the smallest metrics) M_{tails} candidates.

For each extended branch I_{ebr} the conditional expectation of y_{trd} is based on recent symbols history from the root state

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 $S_{root}(i_{ebr})$ and slightly less recent symbol history corresponding tail hanging from $s_{root}(i_{ebr})$ denoted tail (I_{ebr}) . To avoid excess notation, it is assumed in this disclosure that tail (I_{ebr}) contains all the history, since for each tail there is only one root state. Thus the conditional expectation function is denoted $\zeta_{td}(s_{term}(I_{ebr}), tail(I_{ebr}), I_{ms})$.

The metric of each extended branch (I_{ebr}) is minimized over possible parallel transitions (I_{ms}) , and is used to compute the following state metric:

$$\begin{array}{l} M(I_{ebr}) = & \arg\min_{I_{ms}} \|y_{[n]} - \zeta \zeta_{tl}(s_{new}(I_{ebr}), tail(I_{ebr}), I_{ms})\|^2 + \\ M(s_{old}(I_{br})) \end{array} \tag{5}$$

where I_{ebr} is an extended branch index, and there are $N_c \cdot M_{tails}$ possible incoming branches for S_{new} ; tail(I_{ebr}) includes the prior tail of the branch I_{ebr} , which includes all the recent state history $\{a_{[n-k]}\}_{k=D}^{N_h}$ that is relevant for computing metric ("filter memory" and phase); D>0 if it is desired to maintain partial symbol history as explained above; I_{ms} is a parallel transitions indicator, and there are A/N_c possible parallel transitions; and $\zeta_n(s_{new}, \tau, I_{ms})$ is the conditional expectation for $y_{[n]}$ based on state S_{new} , tail τ , and I_{ms} .

Using this formula, an example implementation of the

RSSE circuit 218 using the M-Viterbi algorithm computes $N_c \cdot M_{tails}$ metrics (I_{ebr}) , and choses the M_{tails} tails yielding the smaller (i.e., better) aggregate state metrics as the appropriate survivors for the new $s_{[n]}$ state. At the same time, for each $s_{[n]}$ tail, the RSSE 218 may store the selected extended branch Iebr (providing the minimal metric for the respective $s_{[n]}$ tail). The index of selected extended branch is stored in the traceback memory of the RSSE circuit 218 that indicates, for each pair (state $s_{[n]}$, tail), the selected incoming prior state $s_{[n-1]}$ and tail (i.e. selected extended branch) and also the hypothesized transmitted symbol $a_{[n]}$ at that state $s_{[n]}$. The state index itself implies the coset $a_{[n]}$ % N_c (i.e. the LSBs). Thus, the RSSE circuit **218** may only incrementally store the MSBs per state. Also the traceback memory may not hold traceback data relating to very old information that exceeds the traceback depth discussed below. Thus, the traceback memory may be implemented as a cyclic buffer of depth at least as big as the traceback depth.

Viterbi Traceback

Having updated the state metric for every state in symbol time n, an example implementation of the RSSE circuit **218** implementing the Viterbi algorithm may apply traceback to decode/estimate the transmitted symbols. The traceback depth may indicate the delay of the symbol to be decode with respect to the latest state from which the processing starts (i.e. $s_{[n]}$). This may be, for example, at least 5-10 times the pulse memory, including channel induced ISI. The traceback may be implemented every symbol or every several symbols to reduce complexity.

An example Viterbi traceback operation will now be described. Just after updating all metrics of state set $\{s_{[n]}\}$ the RSSE circuit **218** implementing the Viterbi algorithm finds the best state $s_{[n]}$ based on the aggregate state metrics. Then, using the traceback memory, the RSSE circuit **218** implementing the Viterbi algorithm finds the best prior state for $s_{[n]}$ (denoted $s_{[n-1]}$). This process repeats until reaching the traceback depth (i.e. using the traceback the RSSE circuit **218** implementing the Viterbi algorithm finds for $s_{[n-k]}$ the best prior state $S_{[n-k-1]}$ until $k=N_{tb_depth}-1$). Finally, the traceback process returns the transmission symbol or soft information (e.g., LLRs) attached to

 $^{S}[_{n-N_{tb_depth}}]$

in the traceback memory (i.e. the decoded data).

M-Viterbi Traceback

The traceback in an example implementation of the RSSE circuit **218** implementing the M-Viterbi is similar to as described above when implementing the Viterbi algorithm. For the M-Viterbi algorithm, however, the tails need to be considered in addition to the states.

An example M-Viterbi traceback operation will now be described. Just after updating all metrics of state set $\{s_{[n]}\}$, the RSSE circuit **218** finds the best pair of state and tail (denoted $(s_{[n]}, m_{[n]})$). Then, using the traceback memory the RSSE circuit **218** finds the best prior pair of state and tail denoted $(s_{[n-1]}, m_{[n-1]})$. The process repeats until reaching the traceback depth (i.e. using the traceback the RSSE circuit **218** finds, for $(s_{[n-k]}, m_{[n-k]})$, the best prior pair $(s_{[n-k-1]}, m_{[n-k-1]})$, until $k=N_{nb_depth}-1$. Finally the traceback process returns the transmission symbol or soft information (i.e., LLRs) attached to

$$\left(s_{[n-N_{tb} \ depth]}, m_{[n-N_{tb} \ depth]}\right)$$

in the traceback memory (i.e. the decoded data).

Viterbi Metric Minimization Process

At the input to the Viterbi algorithm (output of mixer 216) $y_{[n]}$ can be modeled as

$$\hat{y}_{[n]} = e^{j\Theta[n]} \cdot h_{pc} * f_{NL}(\Sigma_{k=0}^{N_h} h_{[k]} \cdot a_{[n-k]}), \tag{6}$$

where $a_{[\nu-k]}$ are the previously transmitted symbols; $h_{[1]}$ is the transmit pulse response; $h_{\nu c_{[1]}}$ is the post-cursor ISI that the RSSE circuit **218** implementing the Viterbi algorithm ³⁰ attempts to cancel, where * stands for convolution; $e^{i\Theta_{[\nu]}}$ is the phase rotation due to phase noise; and $f_{NL}()$ is a non-linear function that models the power amplifier (PA) of the transmitter from which the signal was received (e.g., PA **116** when receiving from transmitter **100**). Thus the appropriate branch ³⁵ metric for the Viterbi Algorithm is

$$M_{br}(s_{new}, s_{old}, I_{ms}) = |y_{[n]} - \zeta(s_{new}, s_{old}, I_{ms}) =$$
 (7)

$$=|y_{[n]}-e^{i\theta[n]}\cdot h_{pc}*f_{NL}(\Sigma_{k=0}^{Nh}h_{[k]}\cdot a_{[n-k]})|^{2}=$$
(8)

$$=|y_{[n]} - e^{j.solid} \theta \cdot \Sigma_{l=0}^{N_{pc}} h_{pc[l]} N_{L}(h_{[0]} \mu(S_{new} I_{cs}, I_{ms}) + \sum_{k=1}^{N_{h}} h_{[k]} \operatorname{solid}.a_{[n-l-k]})|^{2}$$
(9)

where S_{new} is the target state for which the RSSE circuit **218** is computing the metric; s_{old} is the designated prior state for S_{new} ; $S_{new}.I_{cs}$ is the coset value that applies (in a fixed way) to the new state s_{new} ; and $s_{old}.a_{[n-k]}$ are the sequence of symbols stored in the prior state s_{old} history for $k <= N_h$; $h_{[-]}$ is the transmit pulse response; h_{pc_1} is the post cursor ISI the RSSE circuit **218** implementing the Viterbi algorithm attempts to cancel, where * stands for convolution; N_{pc} is the length of the post cursor ISI, h_{pc} , that the RSSE circuit **218** implementing the Viterbi algorithm attempts to cancel, assuming that $h_{pc}[0]=1$ is the FFE/DFE cursor; $S_{old,0}$ is the latest phase hypothesis for the prior state s_{old} ; $\mu(l_{cs}, l_{ms})$ is a term that, given coset index and msb selection index, computes symbol value; and l_{ms} the hypothesized MSB bits for $a_{[m]}$.

M-Viterbi Metric Minimization Process

Similarly the appropriate branch metric for the Hybrid M-Viterbi is

$$M_{br}(s_{new}, s_{old}, I_{ms}) = |y_{[n]} - \xi_{tl}(s, \tau, I_{ms})|^2 =$$
 (10) 60

$$=|y_{[n]}^{-}e^{jx\theta}\cdot \sum_{l=0}^{N_{p}} h_{pe[l]}f_{NL}(h_{[0]}^{-}\mu(s.I_{cs},I_{ms}) + \sum_{k=1}^{N_{p}} h_{k[k]}^{-}\tau.a_{[n-l-k]})|^{2}$$
(11)

where τ is a tail corresponding to a prior state of s, $\tau \cdot a_{[n-k]}$ is symbol history of that tail, τ_{θ} is a phase estimate of that tail;

Thus, the M-Viterbi metric is similar to Viterbi metric except for substituting the prior state data s_{old} . $a_{[n-k]}$, $s_{old,\theta}$, by

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the prior tail data $\tau \cdot a_{[n-k]}$, τ_0 . The same substitution can be used in above expressions that are written in terms of state s_{old} instead of tail τ

The branch metric notation can be simplified, and complexity reduced, by denoting the previous PA output estimations as $y_{PA}[n]$. These estimations may be held in memory as part of state s_{old} or tail τ history to avoid any need to recompute them.

$$\begin{split} M_{br}(S_{new},s_{old},I_{ms}) &= |y_{[n]} - e^{i\cdot sold} \Theta(f_{NL}(h_{[0]};\mu) \\ &(S_{new},I_{cs'},I_{ms'}) + \sum_{l=1}^{N_{l}} h_{[k]} \cdot s_{old} \cdot a_{[n-k]}) + \\ &\sum_{l=1}^{N_{p}} h_{pc}[IJ \cdot y_{PA_{[n-l]}})|^{2} \end{split} \tag{12}$$

Based on the branch metric of (12) above, the expectation can be written as shown in (13):

$$\begin{array}{c} \zeta(s_{new},s_{old},I_{ms}) = e^{isold} \theta(f_{NL}(h_{[0]},\mu(S_{new},I_{cs},I_{ms}) + \\ \sum_{k=1}^{N} h_{[k]}s_{old},a_{[n-k]}) + \sum_{l=1}^{N} p_{c}h_{pc}[l] y_{NLPA_{[l}}n - l]) \end{array}$$
 (13)

Parallel Transitions:

As explained previously, to update the state \mathbf{s}_{new} metric, the RSSE circuit **218** implementing the Viterbi algorithm may attempt to minimize every branch \mathbf{I}_{br} metric over all possible parallel transitions \mathbf{I}_{ms} .

$$M(s_{new}I_{br}) = \underset{me}{\operatorname{argmin}} ||v_{[n]} - \xi(s_{new}, s_{old}(I_{br}), I_{ms})||^2 + M(s_{old}(I_{br}))$$
 (14)

Similarly, the RSSE circuit 218 implementing the M-Viterbi algorithm may attempt to minimize the extended branch metric over all possible parallel transitions I_{ms}

$$\begin{array}{ll} M(I_{ebr}) = & \arg\min_{I_{ms}} \|v_{[n]} - \zeta_{tl}(s_{new}(I_{ebr}), tail(I_{ebr}), I_{ms})\|^2 + \\ M(s_{old}(I_{br})) \end{array} \tag{15}$$

In both cases, the RSSE circuit **218** may attempt to minimize a metric over all possible different MSB's (parallel transitions) indexed by I_{ms} . In one embodiment, the metrics are computed per state $s_{[n]}$ for every I_{ms} and then the minimum is computed. However this embodiment has significant computational complexity (A/N_c) .

In another embodiment, in order to reduce complexity, some of the MSBs are determined directly (i.e. by slicing) without the need to compute a metric for each MSB combination. For example, every state $s_{[n]}$ implies a coset for the new symbol $a_{[n]}$, and thus determines the new symbol LSBs. In this manner, the RSSE circuit **218** determines a coset value for the LSBs. With the coset determined, the RSSE circuit **218** may slice the MSBs and compute the metric corresponding to the resulting (sliced) $a_{[n]}$.

For example if the mapper 106 performs integer mapping of the 4 cosets and of the MSB indices, i.e.

$$\mu(I_{cs}, I_{ms}) = 2 \cdot I_{cs} + 4 \cdot I_{ms} - \sqrt{A} + 1 \tag{16}$$

where A is the number of points for the square constellation. Then

$$\begin{cases}
f_{NL}^{-1} \left(\frac{y_{[n]}}{e^{j \cdot s} \text{old} \cdot \theta} - \sum_{l=1}^{N_{pc}} h_{pc}[l] \cdot y_{NL}[n-l] \right) - \\
\sum_{k=1}^{N_{h}} h_{[k]} \cdot s_{okl} \cdot a_{[n-k]} - \frac{2 \cdot I_{cs} - \sqrt{A} + 1}{4}
\end{cases}$$
round

In a similar way, in another example implementation, the RSSE circuit **218** may test **4** hypotheses for the LSB of the parallel transitions I_{ms} for each branch (extended branch) and corresponding coset I_{cs} , and, for each such branch (extended

branch) and each such hypothesis, slice the rest of the MSBs. Finally. The RSSE circuit **218** may select from the hypothesized parallel transitions I_{ms} for a given coset I_{cs} and branch (extended branch), the parallel transition having the lowest metric. The down side of slicing parallel transition MSBs is the need to invert the non-linearity, which may increase noise. However the metric computation to be minimized (i.e. $M(s_{new}, I_{br})$ or $M(I_{ebr})$) does not involve f_{NL}^{-1} and therefore does not increase noise. Thus, as the number of hypotheses of I_{ms} LSBs taken prior to minimizing over all hypothesis per coset per branch metric increases, the probability of error decreases due to complexity reduction.

In another example implementation, in order to reduce the Viterbi memory by 1 (i.e. from N_t to N_t-1) the RSSE circuit **218** may use $y_{[n]}$ and $y_{[n-1]}$ to compute parallel transitions for $a_{[n-1]}$. In this case it may be desirable to account for $y_{[n]}$ being affected by both $a_{[n-1]}$ and $a_{[n]}$. Thus the RSSE circuit **218** recovers $a_{[n]}$ only tentatively to improve slicing of $a_{[n-1]}$ msb's. In this case the RSSE circuit **218** may use several compound hypothesis (e.g. $N_c \cdot N_c$) that include both $a_{[n-1]}$ 20 lsb's and $a_{[n]}$ lsb's. For each such compound hypothesis, the RSSE circuit **218** may slice msb's of both $a_{[n-1]}$ and $a_{[n]}$ in order to get a robust estimation of $a_{[n-1]}$. For each compound hypothesis, the RSSE circuit **218** may compute a metric, and finally select per $a_{[n-1]}$ coset the $a_{[n-1]}$ msb's having best (i.e. 25 lowest) metric.

It should be noted that in order to reduce complexity for relatively flat channel the RSSE circuit **218** may convolve the post cursor response with the composite filter $h_{[\,]}$ and use the trivial post cursor response $h_{p_{C_1}}=[1,0,0,\dots 0]$. In such an 30 implementation, the channel response in the above equations may be rewritten as $h_{[\,]}=\sum_{l=1}^{N_{pc}}h_{pc}[l\,]$. p[n-l].

In FIGS. 1 and 2, busses/data lines labeled with an 'X' operate at the baud rate.

FIG. 3 depicts an example Viterbi implementation of the 35 sequence estimation circuitry of FIG. 2. In FIG. 3, the circuitry 302 convolves the expectancy $\zeta(s_{new},s_{old},I_{ms})$ with the response of shaping filter 108 (FIG. 1) to output signal 303. The circuitry 304 distorts the signal 303 based on a model of the nonlinear distortion present in the signal $y_{[n]}$. The result of 40 the distortion is signal 305. The circuitry 306 convolves the signal 305 with the post-cursor portion of the channel response to generate signal 307, which is an estimation of the signal $y_{[n]}$ given the expectancy $\zeta(s_{new},s_{old},I_{ms})$.

In accordance with an example implementation of this 45 disclosure, an electronic receiver (e.g., 200) comprises frontend circuitry (e.g., 202, 204, 206, 210, 212, 214, and/or 216) and sequence estimation circuitry (e.g., 218). The front-end circuitry is operable to receive a signal over a communication channel, where the received signal is a result of a sequence of 50 symbols being transmitted by a transmitter (e.g., 100). The sequence estimation circuitry is operable to implement a sequence estimation algorithm. In the sequence estimation algorithm, each of a plurality of possible current states of the signal may have associated with it a respective N_c possible 55 prior states and a respective M state extensions, where N_c and M are integers greater than 1. Each iteration of the sequence estimation algorithm may comprise extending each of the plurality of possible current states of the signal by its respective N_c possible prior states and its respective M state exten- 60 sions to generate a respective N_x×M extended states for each of the plurality of possible current states. Each iteration of the sequence estimation algorithm may comprise, for each of the plurality of possible current states of the signal, selecting M of the respective N_c×M extended states to be state extensions 65 for a next iteration of the sequence estimation algorithm. The quantity of states in the plurality of possible states may be less

than the full Viterbi state count. Each of the plurality of possible states may correspond to a sequence of cosets of the symbol constellation QAM used to generate the symbol sequence. The cosets may correspond to one or more least significant bits of a symbol. The sequence estimation circuitry may be operable to, after determination of the least significant bits based on the plurality of metrics, determine most significant bits of the symbol using slicing. The sequence estimation circuitry may be operable to, after determination of the least significant bits based on the plurality of metrics, determine most significant bits of the symbol using a second iteration of the sequence estimation algorithm in which the determined least significant bits are held fixed. The sequence estimation circuitry may be operable to determine a first one or more most significant bits of the symbol using slicing and a second one or more most significant bits of the symbol using a second iteration of the sequence estimation algorithm in which previously determined least significant bits are held fixed. The state extensions may correspond to previous state information that is older than previous state information represented by the plurality of the possible states. The sequence estimation circuitry may be operable to sort the plurality of extended states for each of the plurality of possible states, where the sort is based on the plurality of metrics.

In accordance with an example implementation of this disclosure, an electronic receiver (e.g., 200) comprises frontend circuitry (e.g., 202, 204, 206, 210, 212, 214, and/or 216) and sequence estimation circuitry (e.g., 218). The front-end circuitry is operable to receive a signal over a communication channel, where the received signal is a result of a sequence of symbols being transmitted by a transmitter (e.g., 100). The sequence estimation circuitry is operable to implement a sequence estimation algorithm. The sequence estimation algorithm may comprise, at symbol time n-1 (an arbitrary symbol time): extending a particular possible state of the signal by N_c possible prior states for the particular possible state, resulting in N_c extended states; extending each of the N_c extended states using their extension tails, resulting in N_c×M first extended states with tails; and selecting M of the N_c×M extended states with tails as second state extension tails for the particular possible state. The sequence estimation algorithm may comprise, at symbol time n (the symbol time following the symbol time n-1): generating second extended states with tails using the M second state extension tails.

The present method and/or system may be realized in hardware, software, or a combination of hardware and software. The present methods and/or systems may be realized in a centralized fashion in at least one computing system, or in a distributed fashion where different elements are spread across several interconnected computing systems. Any kind of computing system or other apparatus adapted for carrying out the methods described herein is suited. A typical combination of hardware and software may be a general-purpose computing system with a program or other code that, when being loaded and executed, controls the computing system such that it carries out the methods described herein. Another typical implementation may comprise an application specific integrated circuit or chip. Some implementations may comprise a non-transitory machine-readable (e.g., computer readable) medium (e.g., FLASH drive, optical disk, magnetic storage disk, or the like) having stored thereon one or more lines of code executable by a machine, thereby causing the machine to perform processes as described herein.

While the present method and/or system has been described with reference to certain implementations, it will be understood by those skilled in the art that various changes may be made and equivalents may be substituted without

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departing from the scope of the present method and/or system. In addition, many modifications may be made to adapt a particular situation or material to the teachings of the present disclosure without departing from its scope. Therefore, it is intended that the present method and/or system not be limited to the particular implementations disclosed, but that the present method and/or system will include all implementations falling within the scope of the appended claims.

What is claimed is:

1. A system comprising:

an electronic receiver comprising:

front-end circuitry operable to receive a signal over a communication channel, wherein said received signal is a result of a sequence of symbols being transmitted by a transmitter; and

sequence estimation circuitry operable to implement a sequence estimation algorithm in which:

each of a plurality of possible current states of said received signal has associated with it a respective N_c possible prior states and a respective M state 20 extensions, where N_c and M are integers greater than 1;

for each iteration of said sequence estimation algorithm:

each of said plurality of possible current states of 25 said received signal is extended by its respective N_c possible prior states and its respective M state extensions to generate a respective N_c×M extended states for each of said plurality of possible current states; and

for each of said plurality of possible current states of said received signal, M of said respective N_c×M extended states are selected to be state extensions for a next iteration of said sequence estimation algorithm.

- 2. The system of claim 1, wherein how many states are in said plurality of possible current states is less than a full Viterbi state count.
- 3. The system of claim 1, wherein each of said plurality of possible current states corresponds to a sequence of cosets of 40 a symbol constellation used to generate said sequence of symbols.
- **4**. The system of claim **3**, wherein a coset of said sequence of cosets corresponds to one or more least significant bits of a symbol.
- 5. The system of claim 4, wherein said sequence estimation circuitry is operable to, after determination of said least significant bits based on a plurality of metrics, determine most significant bits of said symbol using slicing.
- 6. The system of claim 4, wherein said sequence estimation 50 circuitry is operable to, after determination of said least significant bits based on a plurality of metrics, determine most significant bits of said symbol using a second iteration of said sequence estimation algorithm in which said determined least significant bits are held fixed.
- 7. The system of claim 4, wherein said sequence estimation circuitry is operable to determine a first one or more most significant bits of said symbol using slicing and a second one or more most significant bits of said symbol using a second iteration of said sequence estimation algorithm in which previously determined least significant bits are held fixed.
- 8. The system of claim 1, wherein said state extensions correspond to prior state information that is older than prior state information represented by said N_c possible prior states.
- 9. The system of claim 1, wherein said sequence estimation 65 circuitry is operable to sort said N_c×M extended states for each of said plurality of possible current states.

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10. A method comprising:

in an electronic receiver:

- receiving, via front-end circuitry of said electronic receiver, a signal over a communication channel, wherein said received signal is a result of a sequence of symbols being transmitted by a transmitter; and
- demodulating, in sequence estimation circuitry of said electronic receiver, said received signal using a sequence estimation algorithm in which:
 - each of a plurality of possible current states of said received signal has associated with it a respective N_c possible prior states and a respective M state extensions, where N_c and M are integers greater than 1; and

each iteration comprises:

extending each of said plurality of possible current states of said received signal by its respective N_c possible prior states and its respective M state extensions to generate a respective N_c×M extended states for each of said plurality of possible current states; and

for each of said plurality of possible current states of said received signal, selecting M of said respective N_c×M extended states to be state extensions for a next iteration of said sequence estimation algorithm.

- 11. The method of claim 10, wherein how many states are 30 in said plurality of possible current states is less than a full Viterbi state count.
 - 12. The system of claim 10, wherein each of said plurality of possible current states corresponds to a sequence of cosets of a symbol constellation used to generate said sequence of symbols.
 - 13. The system of claim 12, wherein a coset of said sequence of cosets corresponds to one or more least significant bits of a symbol.
 - 14. The system of claim 13, comprising:

determining, by said sequence estimation circuitry, said least significant bits based on a plurality of metrics; and

- after said determining said least significant bits, determining, by said sequence estimation circuitry, most significant bits of said symbol using slicing.
- 15. The system of claim 13, comprising:

determining, by said sequence estimation circuitry, said least significant bits based on a plurality of metrics; and

- after said determining said least significant bits, determining, by said sequence estimation circuitry, most significant bits of said symbol using a second iteration of said sequence estimation algorithm in which said determined least significant bits are held fixed.
- 16. The system of claim 13, comprising determining, by said sequence estimation circuit, a first one or more most significant bits of said symbol using slicing and a second one or more most significant bits of said symbol using a second iteration of said sequence estimation algorithm in which previously determined least significant bits are held fixed.
- 17. The system of claim 10, wherein said state extensions correspond to prior state information that is older than prior state information represented by said N_c possible prior states.
- 18. The system of claim 10, comprising sorting, by said sequence estimation circuitry, said $N_c \times M$ extended states for each of said plurality of possible current states.

19. A system comprising:

an electronic receiver comprising:

front-end circuitry operable to receive a signal over a communication channel, wherein said received signal is a result of a sequence of symbols being transmitted 5 by a transmitter; and

sequence estimation circuitry operable to implement a sequence estimation algorithm in which:

- at symbol time n-1, a particular possible state of said received signal is extended by $N_{\scriptscriptstyle C}$ possible prior 10 states for said particular possible state, resulting in $N_{\scriptscriptstyle C}$ extended states;
- at symbol time n-1, each of said N_c extended states is extended by M first state extension tails, resulting in N_c×M first extended states with tails;
- at symbol time n-1, M of said $N_c \times M$ extended states with tails are selected as second state extension tails for said particular possible state; and
- at symbol time n, said M second state extension tails is used for generating second extended states with 20 tails

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